* Finding P.I :

Case-1 :

If $r(x) = e^{ax}$, then.

$$\frac{1}{F(D)} r(x) = \frac{1}{F(D)} (e^{ax}) = \frac{1}{F(a)} e^{ax} \qquad \text{where } F(a) \neq 0$$

If $F(a) = 0$ [case of failure]

$$\frac{1}{F(D)} e^{ax} = x \cdot \frac{1}{F'(a)} e^{ax}, \quad F'(a) \neq 0.$$

Case 2 :

If $r(x) = \sin(ax+b)$ or $\cos(ax+b)$

$$\frac{1}{F(D^2)} \sin(ax+b) = \frac{1}{F(-a)^2} \sin(ax+b), \quad F(-a^2) \neq 0.$$

Case 3: If $r(x) = e^{ax} V(x)$

$$\frac{1}{F(D)} r(x) = \frac{1}{F(D)} [e^{ax} V(x)] = e^{ax} \frac{1}{F(D+a)} V(x)$$

Case - 4:

$$\frac{1}{F(D)}[x v(x)] = x \cdot \frac{1}{F(D)} v(x) + \frac{d}{dD}\left[\frac{1}{F(D)}\right] v(x)$$

Method of variation of parameter:

$$\frac{d^2y}{dx} + P\frac{dy}{dx} + Q y(x) = r(x)$$

Let $y_c = C.F = c_1 y_1 + c_2 y_2$

find $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

Hence. $y_p = -y_1 \int \frac{y_2 r(x)}{W} dx + y_2 \int \frac{y_1 r(x)}{W} dx.$

Method of understermined coefficient

(i) If $r(x) = e^{ax}$ then, $y_p = c_1 e^{ax}.$

(ii) If $r(x) = \cos mx$ or $\sin mx$

$y_p = c_1 \cos mx + c_2 \sin mx.$

(iii) $r(x) = x^m$, then $y_p = a_0 + a_1 x + a_2 x^2 - - - a_m x^m$

(iv) $r(x) = e^{ax} \sin bx$ or $e^{ax} \cos bx$

$y_p = e^{ax}[c_1 \cos bx + c_2 \sin bx]$

Binomial Expansion: $(1+x)^n =$

$$1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)x^3}{3!} - - -$$

Euler - Cauchy Equation [variable parameters]

eg → $x^2y'' + 2y = 2x + 6$

Take $\boxed{x = e^t}$

$$\boxed{xD = \theta \qquad x^2D^2 = \theta(\theta - 1)}$$

$$[\theta(\theta-1) + 2] = 2e^t + 6.$$

$$[\theta^2 - 2\theta + 2] = 2e^t + 6.$$

$A \cdot E = m^2 - 2m + 2$ — —

$$y_c = c_1e^{2t} + c_2e^{2t}.$$

$$y_p = \frac{1}{(\theta^2 - 2\theta + 2)}[2e^t + 6] = \frac{1 \times 2e^t}{(\theta^2 - 2\theta + 2)} + \frac{6\, e^{(0)t}}{(\theta^2 - 2\theta + 2)}.$$