Chapter-1

→ $ydx + xdy = 0$

$M = y \quad N = x$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ = exact

} $\int M dx + \int$ (terms of x not containing x) $dy = C$.

⇒ **Integrating factor:**

(i) Homogenious $M[x,y]$ & $N[x,y]$

$$I.F = \frac{1}{Mx + Ny}.$$

(ii) $f_1(x,y)\,y\,dx + f_2(x,y)\,x\,dy = 0$

$$I.F = \frac{1}{Mx - Ny}$$

(iii) $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ [function of x only]

then $I.F = e^{\int f(x)dx}$

(iv) $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = g(y)$ [function of y-only]

then $I.F = e^{-\int g(y)dy}$

(v) Finding I.F by inspection:

→ $xdy + ydx = d(xy)$

$\frac{xdy - ydx}{x^2 - y^2} = d\left[\frac{1}{2}\log\left[\frac{x+y}{x-y}\right]\right]$

→ $\frac{xdy - ydx}{x^2} = d\left(\frac{x}{y}\right)$

→ $\frac{xdy - ydx}{y^2} = -d\left(\frac{x}{y}\right)$

→ $\frac{xdy - ydx}{xy} = d\left[\log\left[\frac{y}{x}\right]\right]$

→ $\frac{xdy - ydx}{x^2 + y^2} = d\left[\tan^{-1}\frac{y}{x}\right]$

→ taking $\frac{dy}{dx} = p$

eg → $y\left(\frac{dy}{dx}\right)^2 - (x-y)\frac{dy}{dx} - x = 0 \Rightarrow yp^2 - xp + yp - x = 0$

$(p+1)(yp-x) = 0$ } solve individually and multiply it with their final result.

→ Clairaut's equation:

$y = px + f[p] \rightarrow$ ~~$y = px + p$~~ $y = cx + f(c)$

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